Table 1

$-f'(0) \approx \alpha$	$-\beta$ Kennedy	$-\beta$ (19)
0.1637	10^{-2}	0.35×10^{-2}
0.1074	10^{-3}	0.85×10^{-3}
0.0713	10^{-4}	1.15×10^{-4}
0.0513	10^{-5}	1.21×10^{-5}
0.0396	10^{-6}	1.17×10^{-6}

In the Table 1, a comparison is made between Kennedy's numerical results and those predicted by (19).

It may be noted that, for $|\beta| \leqslant 10^{-3}$, the errors in (19) correspond to errors $O(\alpha^2)$ in $f'(0)$ and may well arise from the neglect of α^2 in (7).

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Hydromagnetic Flow between Two Rotating Cylinders

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RAMAMOORTHY[1] has considered the hydromagnetic flow between two concentric rotating cylinders subject to a radial magnetic field. He has used the approximate equation deduced by Rossow,[2] neglecting the induced magnetic field. The simultaneous solution of the Navier-Stokes equations and Maxwell's equations for any hydromagnetic problem presents a formidable task. The induced magnetic field is generally ignored in order to uncouple the two sets of equations. The object of this note is to demonstrate that, in the problem of axisymmetric rotational flow in an annular channel, the two sets of equations do not get coupled even when the problem is generalized so that 1) the induced magnetic field is not ignored, 2) an axial pressure gradient is present, 3) the outer cylinder has a translational velocity in the axial direction besides the uniform rotational velocity, and 4) a uniform suction velocity is imposed at the wall of outer cylinder and a uniform injection velocity on the wall of inner cylinder.

In this generalized problem, it is demonstrated that the principle of independence of axial and rotational field holds. Generalized Couette-type flow caused by the relative motion of the walls of the channel, together with an axial pressure gradient with suction at the walls in the absence of magnetic field, has been considered by various authors.[3–6] The case of flow due to the axial motion of the outer cylinder with suction at the walls and an axial pressure gradient has been studied by Jain and Mehta.[7] These all become special cases of the problem considered here.

We consider steady-state laminar flow of an incompressible, viscous, electrically conducting fluid through an annulus with a and b as its inner and outer radii. Since the annulus is of infinite length in both directions (z axis) with no entry region of the fluid into the annulus, suction rate at the wall of the outer cylinder must be equal to the injection rate at the inner cylinder, i.e., $aV_a = bV_b$, where V_a and V_b are velocities of fluid injection and withdrawal. Cylindrical polar coordinates (r,ϕ,z) are used having the axis of channel as the z axis.

Since the problem is axisymmetric and the cylinders are infinite, the physical variables can be assumed to be independent of ϕ and z. They depend on the radial distance r only.

We assume that

$$\begin{aligned} \mathbf{V} &= [V_r(r), V_\phi(r), V_z(r)] \\ \mathbf{H} &= [H_r(r), H_\phi(r), H_z(r)] \end{aligned} \tag{1}$$

The divergence relation for $\mathbf{V}$ and $\mathbf{H}$ admit of a radial velocity and a radial magnetic field inversely proportional to r, that is,

$$V_r = bV_b/r \qquad H_r = A/r \tag{2}$$

where A is a constant.

Introducing an axial pressure gradient $-\partial p/\partial z = P =$ const, the magnetohydrodynamic equations in mks units take the following form:

Momentum

$$\frac{b^2V_b^2}{r^3} + \frac{V\phi^2}{r} = \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{\rho}\left[\frac{H\phi^2}{r} + H_\phi\frac{dH_\phi}{dr} + H_z\frac{dH_z}{dr}\right] \tag{3}$$

$$\frac{d^2V_\phi}{dr^2} - (R-1)\frac{1}{r}\frac{dV_\phi}{dr} - (R+1)\frac{V_\phi}{r^2} + \frac{A\mu}{\rho\nu}\frac{H_\phi}{r^2} + \frac{A\mu}{\rho\nu}\frac{1}{r}\frac{dH_\phi}{dr} = 0 \tag{4}$$

$$\frac{d^2V_z}{dr^2} - (R-1)\frac{1}{r}\frac{dV_z}{dr} + \frac{A\mu}{\rho\nu}\frac{1}{r}\frac{dH_z}{dr} + \frac{P}{\rho\nu} = 0 \tag{5}$$

Ohm's Law

$$V_\phi = (\nu/A\varkappa)[(\varkappa R - 1)H_\phi - r(dH_\phi/dr)] \tag{6}$$

$$V_z = (\nu/A\varkappa)[\varkappa R H_z - r(dH_z/dr)] \tag{7}$$

where R is suction Reynolds number (bV_b/ν) and $\varkappa = \sigma\mu\nu$.

The boundary conditions are

$$\left.\begin{aligned} V_\phi &= b\omega_2 \\ H_\phi &= 0 \end{aligned}\right\} \text{at } r = b \qquad \left.\begin{aligned} V_z &= U \\ H_z &= 0 \end{aligned}\right\} \text{at } r = b \tag{8}$$
$$V_\phi = a\omega_1 \ \text{at } r = a \qquad V_z = 0 \ \text{at } r = a$$

Equations (4) and (6) are two simultaneous ordinary differential equations in V_ϕ and H_ϕ, and they determine the peripheral velocity and magnetic field. Equations (5) and (7) similarly determine the axial fields. The two pairs of equations can be solved independently of each other. Thus the axial and peripheral fields do not interact in the problem posed. The radial pressure distribution can be determined from Eq. (3) after V_ϕ and H_ϕ are obtained as solutions of Eqs. (4) and (6).

Eliminating V_ϕ between Eqs. (4) and (6) and V_z between Eqs. (5) and (7), we get

$$r^3\frac{d^3H_\phi}{dr^3} + (4 - R - \varkappa R)r^2\frac{d^2H_\phi}{dr^2} + [(R-1)(\varkappa R - 2) - (R+1) - M^2]r\frac{dH_\phi}{dr} + [(R+1)(\varkappa R - 1) - M^2]H_\phi = 0 \tag{9}$$

$$r^3\frac{d^3H_z}{dr^3} + (3 - R - \varkappa R)r^2\frac{d^2H_z}{dr^2} + [(R-1)(\varkappa R - 1) - M^2]r\frac{dH_z}{dr} - \frac{A\varkappa}{\rho\nu^2}r^2P = 0 \tag{10}$$

where $M^2 = A^2\mu^2\sigma/\rho\nu$. The sets of three boundary condi-

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tions to be satisfied for the third-order ordinary differential equations (9) and (10) are

$$\left.\begin{array}{l} \left.\begin{array}{ll} dH_\phi/dr &= -(A\varkappa/\nu)\omega_2 \\ H_\phi &= 0 \end{array}\right\} \text{at } r = b \\ \left.\begin{array}{ll} dH_z/dr &= -U(A\varkappa/\nu b) \\ H_z &= 0 \end{array}\right\} \text{at } r = b \\ \left[(\varkappa R - 1)H_\phi - a(dH_\phi/dr)\right] = (A\varkappa/\nu)a\omega_1 \text{ at } r = a \\ \left[\varkappa R H_z - a(dH_z/dr)\right] = 0 \text{ at } r = a \end{array}\right\} \tag{11}$$

The expressions for V_ϕ, H_ϕ and V_z, H_z are given by

$$(A\varkappa/\nu)V\phi = A_1\varkappa R(1/r) + A_2(\varkappa R - 1 - \lambda_1)r^{\lambda_1} + A_3(\varkappa R - 1 - \lambda_2)r^{\lambda_2} \tag{12}$$

$$H\phi = A_1(1/r) + A_2 r^{\lambda_1} + A_3 r^{\lambda_2} \tag{13}$$

where

$$\lambda_1,\lambda_2 = \tfrac{1}{2}[R(1 + \varkappa) \pm \{R^2(1 + \varkappa)^2 - 4(R + 1)(\varkappa R - 1) + 4M^2\}^{1/2}] \tag{14}$$

$$(A\varkappa/\nu)V_z = \varkappa R B_1 + (\varkappa R - \alpha_1)B_2 r^{\alpha_1} + (\varkappa R - \alpha_2)B_3 r^{\alpha_2} + (\varkappa R - 2)Kr^2 \tag{15}$$

$$H_z = B_1 + B_2 r^{\alpha_1} + B_3 r^{\alpha_2} + Kr^2 \tag{16}$$

$$\left.\begin{array}{l} \alpha_1,\alpha_2 = \tfrac{1}{2}[R(1 + \varkappa) \pm \{R^2(1 - \varkappa)^2 + 4M^2\}^{1/2}] \\ \text{where} \\ K = (AP\varkappa/\rho\nu^2)[(R - 2)(\varkappa R - 2) - M^2]^{-1} \end{array}\right\} \tag{17}$$

and A's and B's are constants of integration. Their values satisfying the boundary conditions (11) become

$$A_1 = (A\varkappa/\nu)(1/c)[\omega_1(\lambda_2 - \lambda_1) \times (a^2b^{1+\lambda_1}b^{1+\lambda_2} - \varkappa Rb^2b^{1+\lambda_1}b^{1+\lambda_2}) + \omega_2\{(\lambda_2 - \lambda_1)\varkappa Rb^2b^{1+\lambda_1}b^{1+\lambda_2} + (\varkappa R - 1 - \lambda_2) \times b^2a^{1+\lambda_2}b^{1+\lambda_1} - (\varkappa R - 1 - \lambda_1)b^2a^{1+\lambda_1}b^{1+\lambda_2}\}] \tag{18}$$

$$A_2 = (A\varkappa/\nu)(1/c)[\omega_1(1 + \lambda_2)(a^2b^{1+\lambda_2} - \varkappa Rb^2b^{1+\lambda_2}) + b^2\omega_2\{\lambda_2\varkappa Rb^{1+\lambda_2} + (\varkappa R - 1 - \lambda_2)a^{1+\lambda_2}\}] \tag{19}$$

$$A_3 = (A\kappa/\nu)(1/c)[\omega_1(1 + \lambda_1) \times (\varkappa Rb^2b^{1+\lambda_1} - a^2b^{1+\lambda_1}) - b^2\omega_2\{\lambda_1\varkappa Rb^{1+\lambda_1} + (\varkappa R - 1 - \lambda_1)a^{1+\lambda_1}\}] \tag{20}$$

where

$$C = \varkappa Rb^{1+\lambda_1}b^{1+\lambda_2}(\lambda_1 - \lambda_2) + (1 + \lambda_2)(\varkappa R - 1 - \lambda_1)a^{1+\lambda_1}b^{1+\lambda_2} - (1 + \lambda_1)(\varkappa R - 1 - \lambda_2)a^{1+\lambda_2}b^{1+\lambda_1}$$

and

$$B_1 = \frac{Kb^2}{D}\left[(\alpha_2 - \alpha_1)\left\{(\varkappa R - 2)(1 - \theta^2) - \frac{A\varkappa}{K\nu b^2}U\right\} - \left(\frac{A\varkappa U}{K\nu b^2} + 2\right)\{(\varkappa R - \alpha_1)(1 - \theta^{\alpha_1}) - (\varkappa R - \alpha_2)(1 - \theta^{\alpha_2})\}\right] \tag{21}$$

$$B_2 = -\frac{K}{Db^{\alpha_1}}\left[\alpha_2(1 - \theta^2)(\varkappa R - 2) - \alpha_2\frac{A\varkappa U}{K\nu} - \left(\frac{A\varkappa U}{K\nu} + 2b^2\right)(1 - \theta^{\alpha_2})(\varkappa R - \alpha_2)\right] \tag{22}$$

$$B_3 = -\frac{K}{Db^{\alpha_2}}\left[(\varkappa R - \alpha_1)(1 - \theta^{\alpha_1})\left(\frac{A\varkappa U}{K\nu} + 2b^2\right) - b^2\alpha_1\left\{(\varkappa R - 2)(1 - \theta^2) - \frac{A\varkappa U}{K\nu b^2}\right\}\right] \tag{23}$$

where

$$D = \alpha_2(\varkappa R - \alpha_1)(1 - \theta^{\alpha_1}) - \alpha_1(1 - \theta^{\alpha_2})(\varkappa R - \alpha_2)$$
$$\theta = a/b$$

The transverse field given by Eqs. (12) and (13) reduces to the one obtained by Ramamoorthy[1] for the case of zero suction velocity at the walls ($R = 0$). The axial field given by Eqs. (15) and (16) agrees with the solution of Jain and Mehta.[7]

In the absence of magnetic field $A = 0$, $M = 0$, $\varkappa = 0$, Eqs. (4) and (5) become exclusive equations in the dependent variables V_ϕ and V_z, respectively. The rotational field reduces to one obtained by Schlichting, and the translational field agrees with Dunwoody[5] and Mehta's[6] results. The field due to simultaneous rotation of the two cylinders and the translation of the outer can be obtained by superposition of the two. It must be noted that the case without the presence of the magnetic field cannot be obtained directly from the solution (12) to (23) by putting $A = 0$, $M = 0$, $\varkappa = 0$. This is accounted for by the fact that, in the case of the hydromagnetic problem, the velocity field is obtained from Eqs. (6) and (7) only after a nonzero magnetic field as a solution of Eqs. (9) and (10) has been determined.

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Thermodynamic Properties of High-Temperature Helium

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Nomenclature

c_2 = second radiation constant = hc/k
E_L = term value of the Lth electronic level, cm^{-1}
f_ι = partition function
g_L = statistical weighting function of the Lth electronic level
h = Planck's constant
I_b = term value of the energy required to ionize the parent atom to the bth degree of ionization, cm^{-1}
k = Boltzmann constant
m_{He} = mass of helium
m_e = mass of electron
T = temperature, °K
V = volume of mixture based on one mole at standard conditions
ρ = density

Subscript

0 = standard conditions (see Table 1)

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